COMPLETENESS OF EXPONENTIAL SYSTEMS

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Let $\Lambda = \{\lambda_n\}$, $n \in \mathbb{N}$, be a sequence of pairwise different complex numbers (points) in the complex plane $\mathbb{C}$, $0 \notin \Lambda$, and $\lambda_n \to \infty$ as $n \to +\infty$.

Let X be a *convex* open or closed set on $\mathbb{C}$. By $H(X)$ denote the space of all continuous complex-valued functions on X which are holomorpic in the interior $\operatorname{Int} X$ of X (if $\operatorname{Int} X \neq \emptyset$) *with the topology of uniform convergence on compact subsets of* X. A exponential system $\operatorname{Exp}_\Lambda = \{\exp(\lambda_n z)\}$ is complete in X if the closure of the linear span of $\operatorname{Exp}_\Lambda$ in $H(X)$ coincides with $H(X)$.

The completeness problem for a set X is to determine *all* such sequences of exponents Λ for which the system $\operatorname{Exp}_\Lambda$ is complete in X. On the background of known results it is natural to look for the solution of the problem in terms of the density of the distribution of the points of Λ and in terms of the geometric characteristics of X. It is natural to subdivide the completeness problem into six cases according to the type of X:

1) $X = [-a, a] \subset \mathbb{R}$, $a > 0$, and $H(X) = C[-a, a]$;
2) $X = (-a, a)$, where $a \in (0, +\infty]$, or $X = \pm[0, +\infty)$;
3) $X = G$ is an *unbounded convex* domain in $\mathbb{C}$, $[0, +\infty) \subset G$;
4) $X = \operatorname{closure} G$, where G is the same as above;
5) $X = G$ is a *bounded convex* domain in $\mathbb{C}$;
6) $X = K$, where K is a *compact convex* set in $\mathbb{C}$, $\operatorname{Int} K \neq \emptyset$;

We shall talk about completeness of exponential systems in accordance with this systematization. Old and new results (including ours) in this direction will be argued. This material is a part of our extensive survey "The completeness of exponential systems and (non-)uniqueness sets: survey" (more than 100 pages). It is in a development stage for publication.

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