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Uniqueness sequences for Bernstein classes of entire functions and the completeness of exponential systems on an interval

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Denote by $\mathbb{N}$, $\mathbb{R}$, and $\mathbb{C}$ the sets of natural, real and complex numbers.

For a segment $I_d \subset \mathbb{R}$ of length d, we denote by $C(I_d)$ and $L^p(I_d)$ the space of continuous functions f with norm $\|f\|_\infty := \sup\bigl\{|f(x)| \colon x \in I_d\bigr\}$ and the space of functions f with finite norm $\|f\|_p := \Bigl(\int_{I_d} |f(x)|^p\, dx\Bigr)^{1/p}$, $p \geq 1$. For $\sigma \in (0, +\infty)$, denote by B_σ^∞ *the Bernstein space* (of type σ) of all entire functions f of exponential type bounded on $\mathbb{R}$ (see [1]).

Let $\Lambda = \{\lambda_k\}_{k\in\mathbb{N}}$ be a point sequence on $\mathbb{C}$ without limit points in $\mathbb{C}$, and all points λ_k are pairwise different (for simplicity). The sequence Λ is an *uniqueness sequence* for B_σ^∞ iff there exists a nonzero function $f_\Lambda \in B_\sigma^\infty$ such that f_Λ vanish on Λ, i.e. $f_\Lambda(\lambda_k) = 0$ for $k \in \mathbb{N}$.

The exponential system $\{e^{i\lambda_k z}\}_{k\in\mathbb{N}}$, $z \in I_d$, is *complete* in $C(I_d)$ ($L^p(I_d)$ resp.) iff the closure of its linear span coincides with $C(I_d)$ ($L^p(I_d)$ resp.). We solve the following problems.

- *Complete description of uniqueness sequences for* B_σ^∞.
- *Criteria of completeness of exponential system* $\{e^{i\lambda_k z}\}_{k\in\mathbb{N}}$ *in* $C(I_d)$ *or* $L^p(I_d)$ *accurate within one or two exponential functions.*

We formulate these new results in terms of the Poisson and Hilbert transforms. Their proofs are unpublished, but announced in [2].

From the criteria of completeness we can obtain many old results on completeness (for example, the famous Berling–Malliavin Theorem on the radius of completeness [3]), and also new results.

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[2] Khabibullin B. N., *Distribution of zero subsequences for Bernstein space and criteria of completeness for exponential system on a segment* // Electronic publication, http://arxiv.org/, arXiv:1104.2683v2 [math.CV]

[3] Koosis P. *The logarithmic integral. V. II,* Cambridge University Press, 1992.