

# On the representation of meromorphic functions as a ratio of holomorphic functions without common zeros

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Let  $\Omega$  be a domain in the complex plane  $\mathbb{C}$  with the boundary  $\partial\Omega$ . We give a general scheme (balayage method) for representation of meromorphic functions on  $\Omega$  as a ratio of holomorphic functions (without common zeros, possibly) with prescribed growth close to boundary  $\partial\Omega$ . We also illustrate this method by new results for concrete situations and problems (special explicit weights, domains etc.) in classical terms. More specifically, denote by  $\text{Hol}(\Omega)$  the space of all holomorphic functions in  $\Omega$ . Let  $H \subset \text{Hol}(\Omega)$ .

We consider the following two problems:

1. *When the meromorphic function on  $\Omega$  can be represented in the form of the ratio of two functions from the space  $H$ ?*
2. *Suppose that a meromorphic function  $f = p/q$  is the ratio of functions  $p, q \in H$ . What as far as possible minimal space  $\widehat{H} \supset H$  should be to guarantee existence of two functions  $p_0, q_0 \in \widehat{H}$  **without common zeros** representing  $f = p_0/q_0$ ?*

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