On the representation of meromorphic functions as a ratio of holomorphic functions without common zeros

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Let Ω be a domain in the complex plane $\mathbb{C}$ with the boundary $\partial\Omega$. We give a general scheme (balayage method) for representation of meromorphic functions on Ω as a ratio of holomorphic functions (without common zeros, possibly) with prescribed growth close to boundary $\partial\Omega$. We also illustrate this method by new results for concrete situations and problems (special explicit weights, domains etc.) in classical terms. More specifically, denote by $\mathrm{Hol}\,(\Omega)$ the space of all holomorphic functions in Ω. Let $H \subset \mathrm{Hol}\,(\Omega)$.

We consider the following two problems:

1. *When the meromorphic function on Ω can be represented in the form of the ratio of two functions from the space H?*

2. *Suppose that a meromorphic function $f = p/q$ is the ratio of functions $p, q \in H$. What as far as possible minimal space $\widehat{H} \supset H$ should be to guarantee existence of two functions $p_0, q_0 \in \widehat{H}$* **without common zeros** *representing $f = p_0/q_0$?*

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