ZERO (SUB)SETS OF HOLOMORPHIC FUNCTIONS AND COMPLETENESS OF EXPONENTIAL SYSTEMS

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Let $\Lambda = \{\lambda_k\}$ be a sequence in the complex plane $\mathbb{C}$. Let Γ denote the Gamma function. Denote by $H\big(D(r)\big)$ the space of holomorphic functions on $D(r) := \{z \in \mathbb{C}\colon |z| < r\}$ with the topology of uniform convergence on compact subsets of $D(r)$. *The radius of completeness* $R(\Lambda)$ in $\mathbb{C}$ is equal to $\sup\big\{r > 0\colon \big\{\exp(\lambda_k z)\big\}$ is complete system in $H\big(D(r)\big)\big\}$. All main results on $R(\Lambda)$ before 2006 can be found in [Kh06]. We use general results from [Kh07I] to estimate of $R(\Lambda)$. A typical result is

Theorem ([Kh07II; Theorem C]). *Let* $\Lambda \subset (0, +\infty)$, $\pm\Lambda := \Lambda \cup \{-\lambda_k\}$;

$$\Lambda(t) := \sum_{0<\lambda_k\le t} 1; \quad \overline{D}_P(\Lambda) := \frac{2}{\pi} \limsup_{y\to+\infty} \int_0^{+\infty} \frac{y}{y^2+t^2}\,\frac{\Lambda(t)}{t}\,dt$$

is the upper Poisson density of the sequence Λ (see [M–R61]). *Then* $(0{,}8472\dots)\cdot\pi\overline{D}_P(\Lambda) = \sqrt{\pi}\big(\Gamma(3/4)\big)^2\overline{D}_P(\Lambda) \le R(\pm\Lambda) \le \pi\overline{D}_P(\Lambda)$.

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