

# ZERO (SUB)SETS OF HOLOMORPHIC FUNCTIONS AND COMPLETENESS OF EXPONENTIAL SYSTEMS

BULAT N. KHABIBULLIN\*

Let  $\Lambda = \{\lambda_k\}$  be a sequence in the complex plane  $\mathbb{C}$ . Let  $\Gamma$  denote the Gamma function. Denote by  $H(D(r))$  the space of holomorphic functions on  $D(r) := \{z \in \mathbb{C}: |z| < r\}$  with the topology of uniform convergence on compact subsets of  $D(r)$ . The radius of completeness  $R(\Lambda)$  in  $\mathbb{C}$  is equal to  $\sup\{r > 0: \{\exp(\lambda_k z)\}$  is complete system in  $H(D(r))\}$ . All main results on  $R(\Lambda)$  before 2006 can be found in [Kh06]. We use general results from [Kh07I] to estimate of  $R(\Lambda)$ . A typical result is **Theorem** ([Kh07II; Theorem C]). Let  $\Lambda \subset (0, +\infty)$ ,  $\pm\Lambda := \Lambda \cup \{-\lambda_k\}$ ;

$$\Lambda(t) := \sum_{0 < \lambda_k \leq t} 1; \quad \bar{D}_P(\Lambda) := \frac{2}{\pi} \limsup_{y \rightarrow +\infty} \int_0^{+\infty} \frac{y}{y^2 + t^2} \frac{\Lambda(t)}{t} dt$$

is the upper Poisson density of the sequence  $\Lambda$  (see [M–R61]). Then  $(0,8472\dots) \cdot \pi \bar{D}_P(\Lambda) = \sqrt{\pi}(\Gamma(3/4))^2 \bar{D}_P(\Lambda) \leq R(\pm\Lambda) \leq \pi \bar{D}_P(\Lambda)$ .

- [Kh06] B. N. Khabibullin, *Completeness of exponential systems and uniqueness sets. Survey*, Bashkir State University, Ufa, 2006, 188 pages (in Russian).
- [Kh07I] \_\_\_\_\_, “Zero sequences of holomorphic functions, the representation of meromorphic functions, and harmonic minorants”, *Matem. Sbornik*, **198** : 2 (2007), 121–160 (in Russian).
- [Kh07II] \_\_\_\_\_, “Zero sequences of holomorphic functions, the representation of meromorphic functions, and harmonic minorants. II. Entire functions”, submitted to *Matem. Sbornik*.
- [M–R61] P. Malliavin, L. A. Rubel, “On small entire functions of exponential type with given zeros”, *Bull. Soc. Math. France*, **89** (1961), 175–206.

BASHKIR STATE UNIVERSITY, FRUNZE STR. 32, UFA, RB, 450074, RUSSIA  
*E-mail address:* Khabib-Bulat@mail.ru  
*URL: (eng; rus)* [http://math.bsunet.ru/khb\\_e](http://math.bsunet.ru/khb_e) ; [//math.bsunet.ru/khb](http://math.bsunet.ru/khb)

---

1991 *Mathematics Subject Classification*. Primary 30E10; 30D15, 33B15, 31A05.  
*Key words and phrases*. completeness, exponential system, entire function, Gamma function, zero set, uniqueness set, balayage, (sub)harmonic minorant.

\*Supported by RFBR, grant no. 06-01-00067, and by the program “The State Support of Leading Scientific Schools of the Russian Federation”, NSh–10052.2006.1.