Variations of entire (subharmonic) function under perturbations of its zero set (Riesz measure)

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In the "Disribution of zeros of entire functions" (Ch. II, Lemmas 1, 4) B. Ya. Levin studies how shifts in the arguments of the zeros of an entire function influence the growth of this function. His results were generalized by A. A. Gol'dberg (1965) and I. F. Krasichkov-Ternovskiĭ (1966). In 1969 V. S. Azarin gives a general subharmonic interpretation for the concept of shift in the zeros of an entire function and obtains also a new result.

Let T be a Borel-measurable mapping of the complex plane $\mathbb{C}$ into itself. Given Borel measure ν on $\mathbb{C}$, the mapping T generates T-shift ν_T such that $\nu_T(G) := \nu(T^{-1}G)$ for any Borel set G. Below any subharmonic function with Riesz measure ν (ν_T resp.) is denoted by u^ν (u^ν_T resp.).

In terms of uper estimates for $\bigl|1 - T(z)/z\bigr|$ on $\mathbb{C}$, a complete investigation of smallness $|u^\nu - u^\nu_T|$ was realized by B. N. Khabibullin (1984).

In our talk, we shall consider now integral conditions on T of the form

$$\int_{\mathbb{C}\setminus\{z\in\mathbb{C}\colon\, |z|<1\}} \Bigl|1 - \frac{T(z)}{z}\Bigr| \frac{\mathrm{d}\nu(z)}{\Delta(z)} < +\infty, \quad \liminf_{z\to\infty} \Bigl|\frac{T(z)}{z}\Bigr| > 0. \qquad (*)$$

Theorem (for a special case). *Let ν be a measure of finite type for a order $\rho > 0$. If the relation $(*)$ holds for $\Delta(z) \equiv |z|^\rho$, $z \in \mathbb{C}$, then for every number $\varepsilon > 0$ and u^ν there are a constant C_ε and u^ν_T such that*

$$u^\nu_T(z) \le \sup\bigl\{u^\nu(w)\colon |w - z| \le \varepsilon|z|\bigr\} + \varepsilon|z|^\rho + C_\varepsilon, \quad z \in \mathbb{C}.$$

Corollary. *Let f be a entire function with zero set $\{\lambda_n\}$ and indicator function $h_f < \infty$ for a order $\rho > 0$. If $\sum_{\lambda_n\neq 0} |1 - \gamma_n/\lambda_n| \cdot |\lambda_n|^{-\rho} < \infty$, then there is an entire function g with zero set $\{\gamma_n\}$ such that $h_g = h_f$.*

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